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Surface Waves Generated by Atmospheric

Nuclear Explosions

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P. D. Marshall

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Surface Waves Generated by Atmospheric Nuclear Explosions.

E.W. Carpenter P.D. Marshall

Summary

An approximate theoretical analysis is given for Rayleigh waves generated by atmospheric nuclear explosions. An analysis is made of the seismic recordings of Russian explosions at Novava Zemlya and quantitative agreement with the theory is found. Values for the absorption coefficient of the earth are derived and proposals made for a wider use of the stationary phase approximation.

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1. INTRODUCTION

During 1961 and 1962 there were many atmospheric nuclear explosions which were sufficiently large to generate recordable seismic surface waves. Our interest in these waves was stimulated by the installation of a long period vertical seismometer, and it became common practice to examine the seismic traces for the precursors of the atmospheric gravity waves which were recorded on a microbarograph. It was soon obvious that the seismic waves from a given area were remarkably similar in shape, and their amplitude was related empirically (via indendent evidence) to the weapon yield. In the initial stages the Wolverton system was not adequately calibrated and many other seismic records from standard seismic stations distributed around the world were collected and examined.

The accumulation of seismic data and yield estimates from other techniques was soon sufficiently encouraging to justify a theoretical approach to the seismic problem.

2. THEORETICAL ANALYSIS FOR AN HOMOGENEOUS EARTH

2.1 The Surface Point Source Problem

As an introduction to the notation, consider an homogeneous and isotropic half space defined by it's Lamé elastic constants λ and μ and its density ρ and let a harmonic surface point force \dot{Q} exp(iwt) act at the origin. Lamb [1] showed that at a distance R the vertical Rayleigh wave amplitude at the surface. W, is given by the equation

$$W(R) = \frac{Qk^2\xi_0 \alpha_0}{2\mu F'(\xi_0)} \left[\frac{2}{\pi\xi_0 R}\right]^2 \exp i(wt - \xi_0 R + 3\pi/4). \quad (1)$$

1. H. Lamb: (1904) "On the Propagation of Tremors over the Surface of an Elastic Solid". Phil. Trans. Roy. Soc. (London), A203, 1 - 42 In this equation the following notation is employed:-

$$C_{\alpha}^{2} = (\lambda + 2\mu)/\rho \qquad C_{\beta}^{2} = \mu/\rho \\ h^{2} = w^{2}/C_{\alpha}^{2} \qquad k^{2} = w^{2}/C_{\beta}^{2} \\ a^{2} = \xi^{2} - h^{2} \qquad \beta^{2} = \xi^{2} - k^{2} \\ a_{0}^{2} = \xi_{0}^{2} - h^{2} \qquad \beta_{0}^{2} = \xi_{0}^{2} - k^{2} \\ F(\xi) = (2\xi^{2} - k^{2})^{2} - 4\alpha\beta\xi^{2} \end{cases}$$

$$F'(\xi_{0}) = \left[\frac{dF}{d\xi}\right]_{\xi} = \xi_{0}$$

and ξ is the root of $F(\xi) = 0$.

It will be seen that C_{α} and C_{β} are respectively the velocities of compressional and shear waves in the solid while ξ is the Rayleigh wave number which can be written w/C γ , where C γ is the velocity of Rayleigh waves.

If we now make the usual assumption, reasonably valid for most rocks, that $\lambda = \mu$ a considerable numerical simplification can be achieved. Thus, after some arithmetic, we derive

$$W(w,R) = \frac{0.073Q\xi_0^{\frac{1}{2}}}{R^{\frac{1}{2}}\mu} \exp i(wt - \xi_0R + 3\pi/4). \qquad \dots (2)$$

In general the force acting at the origin is not harmonic but will be of the general form B(t). We now use the Fourier transforms

$$B(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} A(w) e^{iwt} dw,$$
 (3)

$$A(w) = \int_{-\infty}^{+\infty} B(t) e^{-iwt} dt, \qquad \dots (4)$$

noting that the function B(t) can be synthesised from the harmonic solution. Thus we have that for a force B(t) acting at the origin

$$W(t,R) = 0.0116 \int_{-\infty}^{+\infty} \frac{A(w)\xi_{0}}{R^{2}\mu} \exp i(wt - \xi_{0}R + 3\pi/4)dw. (5)$$

2.2 The Spherically Expanding Pressure Pulse

In this section we shall retain the notation of the previous section but in addition include a fluid atmosphere above the half space. Neglecting gravity, the atmosphere is considered as an homogeneous fluid defined by its density σ and sound velocity v.

Let a harmonic point compressional source be located at a height D above the origin. Assuming that propagation is linear, the pressure pulse at (radial) distance r in a homogeneous atmosphere can be represented by the equation

$$P(r) = \frac{P_o r_o e^{iwt}}{r}$$
 (6)

The problem of the generation of Rayleigh waves by such a point source in a fluid over a half space has been treated by Ewing, Jardetsky and Press [1] and using their work we derive, after some algebraic manipulation, an expression for the vertical motion at the surface, viz,

$$W(R) = \frac{2P_{o}e^{iwt}r_{o}k^{2}}{\mu} \int \frac{a\xi e^{-\delta D}J_{o}(\xi R)}{\delta F(\xi) + \sigma \rho w^{4}a} d\xi, \qquad \dots (7)$$

$$\delta^{2} = \xi^{2} - (w/v)^{2}.$$

where

The pole contribution from the above equation, which gives the surface wave, is

$$W(w,R) = \frac{2\pi i P_o r_o k^2 a_o \xi_o e^{-\delta_o D_H^2}(\xi_o R) e^{iwt}}{\mu \delta_o \Delta'(\xi_o)}, \qquad \dots (8)$$

where ξ_0 is the root of the equation

$$\Delta(\xi) = F(\xi) + \frac{\alpha \sigma \rho w^4}{\delta} = 0$$

1. W.M. Ewing, W.S. Jardetsky and F. Press: (1957) "Elastic Waves in Layered Media". p.105, McGraw-Hill

$$\Delta'(\xi_{0}) = \left(\frac{d\Delta}{d\xi}\right)_{\xi_{0}}$$

$$a_{0}^{2} = \xi_{0}^{2} - h^{2}, \ \delta_{0}^{2} = \xi_{0}^{2} - w^{2}/v^{2}.$$

As in the previous section $\zeta_0 = w/C_\gamma$, where C_γ is the velocity of Rayleigh waves. In this case [1], however, the Rayleigh pole is not a real number. For the actual case under consideration it seems reasonable to make the assumption $\sigma = 0$, in which case ζ_0

has the same value as in the half space calculation of the previous section.

Using the asymptotic expansion for the Hankel function [2]

$$H_{o}^{(2)}(x) = \left[\frac{2}{\pi x}\right]^{\frac{1}{2}} \exp i(\pi/4 - x),$$

we therefore derive that for a source radiating as $\frac{P_o e^{iwt} r_o}{r}$

$$W(R) \approx \frac{2\pi P_{o} r_{o}^{k^{2} \alpha_{o} \xi_{o}}}{\mu \delta_{o}' F'(\xi_{o})} \left[\frac{2}{\pi \xi_{o} R}\right]^{\frac{1}{2}} \exp i(wt - \xi_{o} R + \pi/4 - \delta' D), \quad (9)$$

where $\delta'_0^2 = -\delta_0^2$ and is essentially positive for the cases of interest, where $C_{\gamma} > v$.

Comparing the modulus of the expression with that for the previous case of a point surface source Qe^{iwt} , we note that $Q \approx \frac{4\pi P_o r_o}{\delta'_o}$, which for $v < C_\gamma$ becomes $Q \approx \frac{4\pi P_o r_o v}{w}$ (10) Note however the phase shift of $\frac{\pi}{2}$.

- 1. L. Cagniard: (1962) "Reflection and Refraction of Progressive Seismic Waves". p.224, Translated by E.A. Flynn and C.H. Dix, McGraw-Hill
- 2. W.M. Ewing, W.S. Jardetsky and F. Press: (1957) "Elastic Waves in Layered Media". p.137, McGraw-Hill

Retransforming into the time domain we have

$$w(t,R) = \frac{0.15r_{0}V}{R^{2}\mu C_{\gamma}^{2}} \int_{-\infty}^{+\infty} P_{0}(w)w^{-2} \exp i(wt - \xi_{0}R + \pi/4 - \delta_{0}'D)dw, \dots (11)$$

and our problem is now to evaluate this integral.

Before continuing, it is pertinent to note that the <u>amplitude</u> of the Rayleigh wave is independent of the height of the source, at first sight a somewhat surprising result. In fact the height of the source only enters the problem through the phase term.

2.3 The Stationary Phase Approximation

The expressions derived for the Rayleigh waves at long ranges are of the form

$$\int_{-\infty}^{+\infty} A(w)f(\xi_0) \exp i(wt - \xi_0 R - \varphi)dw.$$

The evaluation of this integral would give the shape of the wave as a function of time for the homogeneous model considered, but this would have little value since the earth is not homogeneous.

If we confine attention to the period range 10 - 60 s, which is the range within which the observed Rayleigh waves occur, then it is a reasonable first approximation to assume that the earth is composed of plane horizontal layers having different elastic constants. In this case the propagation of Rayleigh waves is characterised by dispersion, i.e., C_y is a function of w.

To enable an order-of-magnitude calculation to be performed we shall assume that the layering has no effect on the amplitude of the signal generated at the source, but that it does cause dispersion, i.e., we assume C_{ν} to be constant in deriving equation (11) and then allow it to be a function of w in evaluating it.

Consider a portion of a dispersed wave train (frontispiece) where the apparent frequency (that obtained by measuring the time between successive peaks) is w_1 and the amplitude is $W(w_1)$. Then by applying the method of stationary phase [1] to evaluate the integral in equation (11), we have

$$W(w_{1}) = 0.37 \frac{\overline{P}_{o}(w_{1}) vr_{o}}{R\mu} \frac{(\xi_{o})^{2}}{w_{1}} \left[\frac{d^{2} \xi_{o}}{dw^{2}}\right]^{-2}, \qquad \dots (12)$$

where $P_0(w_1)$ is the amplitude of the Fourier component of the pressure wave at the frequency w_1 .

To convert this expression into one involving more readily recognisable quantities we note that

$$\left[\frac{d^{2}\xi_{o}}{dw^{2}}\right]_{1} = \frac{d}{dw} \left[\frac{1}{U_{1}}\right]$$

$$= \frac{1}{U_1^2} \left[\frac{dU}{dw} \right]_1$$

$$= \frac{T_1}{2\pi U_1^{2.}} \begin{bmatrix} \frac{dU}{dT} \end{bmatrix}_1,$$

where T is the period of the wave, $T = \frac{2\pi}{w_1}$, and $U_1 = \left[\frac{dw}{d\xi_0}\right]_1$ is the group velocity of the frequency component w_1 .

Thus W(w₁) =
$$\frac{0.37\bar{P}_{o}(w_{1})r_{o}vU_{1}}{R_{\mu}C_{\gamma}^{\nu}T_{1}^{\nu}} \left[dU \\ dT \right]^{\nu}$$
(13)

is an expression which enables the Rayleigh wave amplitude to be estimated if $P_0(w_1) r_0$ can be obtained.

1. W.M. Ewing, W.S. Jardetsky and F. Press: (1957) "Elastic Waves in Layered Media". p.367, McGraw-Hill

2.4 Implications of the Theoretical Analysis

The preceding analysis enables the following conclusions to be drawn. The amplitude of Rayleigh waves generated by atmospheric nuclear explosions can be considered as the product of four terms

$$W(w) = Y(w) \times S(w) \times J(w) \times \psi. \qquad \dots (14)$$

The factor Y(w) depends upon the explosion and is given by $r_{o}\overline{P}(w)$. Linearity has been assumed and its validity has yet to be considered but, assuming linearity, there is no dependence on the height of burst.

The factor S(w) depends upon the local geological conditions. In general, because of layering, S(w) will be a function of the frequency but in our simple approach we have considered an homogeneous earth and the site factor can be written $\frac{0.37v}{\mu C_{\gamma} k}$, where μ and C γ are a kind of weighted average of the crystal parameters. Typical values are estimated as $\mu = 3 \times 10^{11}$ c.g.s. units and C $\gamma = 3.6 \times 10^5$ cm/s, while $v = 3.3 \times 10^4$ cm/s. Thus, S $\approx 0.7 \times 10^{-10}$ c.g.s. units.

The factor J(w) depends upon the transmission path and is given by $\frac{U}{T^{\frac{1}{2}}} \left[\frac{dU}{dT} \right]^{-\frac{1}{2}}$. A typical value at T = 40 s for a continental path is 1200 c.g.s. units.

The factor ψ depends on the distance between source and station. In our plane layer calculation $\psi = 1/R$, but in practice we must allow for a spherical earth and write

$$\Psi^{-1} = E\Delta^{\prime 2} \sin^{\prime 2} \Delta,$$

where E is the radius of the earth and Δ is the angle subtended at the centre of the earth by the source and station.

One other factor can be taken into account and this is absorption, whose effect can be allowed for by inserting a term exp ($-\pi E\Delta/QUT$). in the expression for ψ , where Q is the absorption coefficient.

These considerations lead us to define two programmes of investigation.

For any given station and firing site S, J and i are constant and the amplitude of specific frequency components should depend only upon yield, provided that the linearity approximation can be justified.

For a specific explosion, Y(w) and S(w) are constant, J(w) can be derived from each record if the firing site and time of origin are determined, and $\frac{1}{3}$ is known for each record. Thus, $W(w)/\frac{1}{3}J(w)$ should be a constant for all stations except for the effects of absorption and violations of the assumption of travel path uniformity. A study of $W(w)/\frac{1}{3}J(w)$ will therefore be of considerable geophysical value.

3. THE ATMOSPHERIC PRESSURE PULSE

3.1 The Pressure-Time Profile and Scaling

To a good degree of approximation the pressure pulse from an M megaton explosion in an homogeneous atmosphere can be represented by the equation

$$P(r,t) = P_{0}(r/M^{\frac{1}{3}})(1 - t/M^{\frac{1}{3}}\tau) \exp(-t/M^{\frac{1}{3}}\tau). \qquad \dots (15)$$

For an NTP atmosphere, Figure 1 (derived from data by Glasstone [1]) shows the variation of P and τ with r/M³. If the pulse propagation were a linear process rPM^{-1/4} and τ would both be constant and independent of r. Clearly they are not and we have to make appropriate approximations to linearise the problem.

For large values of $r/M^{\frac{1}{3}}$ both parameters do tend asymptotically to constant values and if the generation of surface waves took place at such distances we could neglect the non-linear effects. Unfortunately this hardly seems likely. There is an approximate rule of thumb that weapons are often fired at heights of about $1.5M^{\frac{1}{3}}$ kilometres, and it is clear from Figure 1 that for such heights of burst, the blast wave is still behaving very non-linearly when it hits the ground.

3.2 The Equivalent Linear Source

Our approach has been to obtain the values of P and τ appropriate to the horizontal range R, where the generation of surface waves is at its maximum. To obtain this distance we have relied on evidence from small-scale model experiments (Appendix), which indicated that for practical purposes the waves are generated within a circle of diameter equal to the height of burst. Simple geometrical

^{1.} S. Glasstone (Editor): (Revised Edition 1962) "The Effects of Nuclear Weapons". USAEC

considerations show that effectively we need only consider the pressure wave at a distance equal to the height of burst and assume that its behaviour is linear over a small (radial) distance range.

Thus, assuming that all explosions take place at the same scaled height, we can assume that the waveform is represented by the Friedlander expression

$$P(\mathbf{r},t) = \frac{\Pr[o'o']{N'_3}}{r} (1 - t/M'_3\tau) \exp(-t/M'_3\tau), \qquad \dots (16)$$

where, for $D = 1.5M^{\frac{1}{3}}$ kilometres, $\tau = 1$ second and $P_{oo}r \approx 4 \times 10^{\frac{1}{2}}$ c.g.s. units (an allowance having been made for non-linear reflection effects).

In the frequency domain we have

$$r\bar{P}(r,w) = \frac{\Pr_{o}r_{o}^{Mw\tau^{2}}}{1 + w^{2}\tau^{2}M^{2/3}} \qquad \dots (17)$$

and from equation (14) we see that this expression shows how the amplitude of a given frequency component at a given station varies with the yield of explosions at a particular test site, assuming that all of the explosions take place at the same scaled height of burst. To illustrate the effect of varying the height of burst Figure 2 has been prepared to show the variation of the numerator variable $\Pr_{\sigma} \tau^2$. It will be seen from this figure that in fact $\Pr_{\sigma} \tau^2$ is fairly constant over a wide range of heights of burst, while from Figure 1, τ varies almost linearly with height of burst. However, τ only occurs in the denominator in combination with other terms and it is only when $M^{2/3} \tau^2 w^2$ is of order unity, or larger, that the denominator is important. It should be noted that the Friedlander waveform is inaccurate for $r/M'_3 < 1 \text{ km/M}'_3$.

In practice we have found it convenient to measure the amplitudes of the records at the points where apparent periods, given by twice the time between successive zero crossings, are 20, 30 and 40 s. In Figure 3 are plotted the predicted variation of amplitude versus yield at these periods for explosions at a height of $1.5M\frac{1}{3}$ kilometres.

One additional point connected with the non-linear behaviour of the pressure pulse is that its velocity is some 40% higher than the velocity of sound waves. This effect can be taken care of by substituting the appropriate value in the expression for S which becomes 10^{-10} c.g.s. units.

As the (absolute) height of the explosion increases other effects connected with the non-uniformity of the atmosphere become increasingly important.

The scaling laws can be modified to take account of variations in ambient pressure and sound speed (temperature) by employing the general equation

$$\frac{P}{P_a} = f(\frac{r}{\alpha}, \frac{t}{\tau^*})$$

in which $\alpha = \left[E/P_a\right]^{l_3}$ and $\tau^* = \alpha/C_a$, where P_a and C_a are the ambient pressure and sound speed and E is the energy release. However, making what are generally small corrections by using the pressure and sound speed at the height of burst when the subsequent propagation is through a non-uniform atmosphere seems of limited value. As the height of burst increases the fraction of the total energy release which becomes available to the pressure wave decreases. This effect is, however, unlikely to be significant for heights of burst below 50 km. We conclude that our overall description of the blast wave is sufficiently accurate for explosions fired at heights less than say 10 km.

4. SUMMARY OF THEORETICAL PREDICTIONS

By combining the estimates of the various parameters we can derive an expression for the amplitude of a particular crest of apparent period T in the dispersed wave train resulting from an atmospheric explosion of M megatons. Making the appropriate substitutions we obtain

$$A(T) \approx \frac{4 \times 10^{-3} M}{(1 + 40M^{2/3}T^{-2}\tau^2)} \left[\frac{U}{T^{3/2}} \left(\frac{dU}{dT} \right)^{-\frac{1}{2}} \right] (18)$$
$$\times \frac{\exp(-\pi E\Delta/QUT)}{\Delta^2 \sin^2 \Delta} \text{ microns,}$$

where $\tau = 1.0$ s for a height of burst of $1.5M^{\frac{1}{3}}$ km and varies almost linearly with height over a range of $M^{\frac{1}{3}}$ to $3M^{\frac{1}{3}}$ km.

As an illustration we will take the example of a 40 s wave recorded at $\Delta = 60^{\circ}$ from a 60 megaton explosion fired at a height of 6 km. Taking representative values

U = 4 km/s
Q = 200
and
$$\frac{U}{T^{2/2}} \left(\frac{dU}{dT}\right)^{-1/2} = 30$$

we obtain A = 2.2 microns.

The average value for the reported 60 megaton shot recorded at six stations was 2.6 microns. The agreement, though remarkable, must be regarded as fortuitous in view of the many approximations. Nevertheless, the result would appear to justify the theoretical approach, and the succeeding sections will describe the analysis of the experimental results on the basis of the theory.

5. EXPERIMENTAL AMPLITUDE DATA

5.1 Initial Analysis

Long period records of the various explosions were collected from many seismic stations throughout the world. These records all have the general form of a dispersed wave train (see frontispiece), the amount of dispersion depending upon the distance of the station from the source and on the character of the propagation path.

For our first analysis we measured the peak-to-peak amplitude of the cycles whose apparent periods, measured between crossings of the zero line, were 20 and 40 s. These amplitudes, in mm, were then converted to microns ground displacement by dividing by the magnification at the appropriate period. Herein lay one of the greatest difficulties, for the magnification data available were often very poor. This is illustrated in Figure 4 which shows the transient response of several of the World Wide Standard Seismographs whose transient responses should be identical. Table 1 lists all the seismic stations from which records were obtained, and gives the magnification figures which we have used. Hence, any subsequent changes in magnification data can be readily assessed. Many of the seismic traces were very noisy despite their low magnifications, an effect which could have been caused by a lack of compensation for atmospheric pressure variations, since we often saw the effect of the long period atmospheric waves generated by the explosions. A typical seismic trace, for PNT, gave amplitudes of 0.5 mm with a trace width of 1.0 mm to be measured against a noise amplitude of 0.3 mm.

Let C(I,J) be the logarithm of the amplitude of a particular period wave recorded at station J from the I^{II} explosion at a particular site. If we assume that a given series of explosions all take place at the same point and that each recording instrument retains its characteristics throughout the series, then we can write

$$C(I, J) = B(J) + A(I) + \varepsilon(I, J),$$
 (19)

where B(J) is an explosion factor, A(I) is a site factor and ε is the error.

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These equations of condition can be solved by the least squares method subject to the condition that the sum of the squares of the error terms is minimised. A programme to carry out the computation was prepared by H. Somers. This also allowed for weights W(I, J)to be associated with each equation, the weight used (1,2,3 or 4) depending primarily on the signal-to-noise ratio.

In fact the solution of the equations is not unique because if we subtract an aribtrary constant from A(I) and add it to B(J) the equations are still satisfied, i.e.,

$$C(I,J) = \left[B(J) - m\right] + \left[A(I) + m\right] + \varepsilon(I,J).$$

To remove this indeterminancy we arbitrarily took A(ALERT) equal to zero.

The procedure described was applied to both the 1961 and 1962 Russian series at Novaya Zemlya. The results for A(I) and a few B(J) together with their standard deviation, σ , and the number of readings employed, N, are given in Tables 2 and 3.

Had stations of the quality of those now operating been routinely operating during the appropriate periods, we are confident that the accuracy of the analysis would have been very much increased. In some cases station bulletins were read to obtain amplitude data. In all cases they were special high quality US stations referred to in Table 1 by the four letter code. A few of these stations gave erratic results, but others gave highly consistent results, a tribute to conscientious routine record analysis.

5.2 Amplitude Yield Analysis

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Assuming that the firing site is essentially the same for all explosions in a given region the values of B(J) give information about the relative yields of the explosions. To obtain absolute yields we could use the approximate calculation of Section 4 but this would place far too much reliance on what was only an order of magnitude calculation. Alternatively we can calculate yield estimates on the basis of Kruschev's statement that the yield of the 30th October 1961 explosion was 60 megatons.

Unfortunately the 1961 series was not particularly well recorded, but, as an example, consider the explosion of 23rd October. At 40 s period the mean ratio between the amplitudes of the waves from 30th October and 23rd October is 2.45 ± 0.10 , while at 20 s the ratio is 2.20 ± 0.05 . Using the theoretical curves for $\tau = 1$ s we obtain M = 22 ± 2 for the 40 s wave, M = 17 ± 2 for the 20 s wave. This discrepancy could be resolved in several ways, in particular the most probable reason is that the nominal 60 megaton shot was fired at a scaled height less than that for the 20 megaton shot.

It is unfortunate that the only yield information released is in the higher yield range where the deviations from linearity are the greatest.

The 1962 Russian series was well recorded at many stations. In principle it is possible to still use the 1961, 60 megaton explosion as a standard of reference, if we can assume that the station magnifications remained constant. Examination of the station terms for ALE, HAL, MNT, RES and WOL indicates that they in fact retained constant gain. Taking a mean of the 20 s station differences for these stations we conclude that to obtain the best consistency between the two series 0.008 should be added to the 20 s B(J) terms for 1961.

It is interesting to note that the consistent differences between the A(J) terms for the 1961 and 1962 series (Uppsala for instance) implied variations in magnification which were often subsequently confirmed. The 1962 American Pacific explosions, although apparently smaller yield devices, were also recorded and when yield data become available it will be possible to compare the Pacific and Novaya Zemlya site factors.

5.3 Analysis of A(I), Amplitude Versus Distance

The terms A(I) give the relative amplitudes recorded at the various stations for a given event, and we are led to examine whether or not these terms are consistent with the picture given in the theoretical section.

The terms A(I) given in Tables 2 and 3 have all been derived from the least squares solution. Many records were obtained from stations where only one event was recorded at sufficient amplitude to enable an accurage measure of C(I,J) to be made. In these cases we have used the appropriate B(J) for the event from Tables 2 and 3 and calculated A(I) directly by subtraction (equation (19)) assuming thereby that $\varepsilon(I,J) = 0$. Additional values of A(I) thus determined are given in Table 4 which summarises all the appropriate data.

According to the theoretical treatment

$$A(I) - \log \left[\frac{U}{T^{3/2}} \left(\frac{dU}{dT} \right)^{-\frac{1}{2}} \right] + \log \left(\Delta^{\frac{1}{2}} \sin^{\frac{1}{2}} \Delta \right) \qquad \dots (20)$$
$$= \frac{\pi \Delta E}{2.3 \text{QUT}} + \text{const.}$$

Now $\Delta^{\prime 2} \sin^{\prime 2} \Delta$ is readily obtained from the co-ordinates of the station and firing site, and both U and $\left(\frac{dU}{dT}\right)^{-\prime 2}$ can be obtained directly from the record, assuming that the travel time is known (see Section 6). As will be clear later from the group velocity curves, dU/dT is rather difficult to measure at periods of both 20 and 40 s. A period of 30 s gives more accurate results and the analysis was therefore extended to include data at this period. All the appropriate values are shown in Table 4, and Figures 5, 6 and 7 show the values of the left hand side of equation (20) plotted against E Δ/U . Figures 5 - 7 were interpreted to give values of Q by fitting a least squares straight line through the points, and the following results were obtained:-

Period c	0	95% Confi	dençe Limits
1 21100, 5	Ŷ	Upper	Lower
40 30 20	400 290 400	∞ 770 700	190 180 290

The result obtained agrees well with contemporary estimates of an average Q for the crust and upper mantle. Since the method allows that scattering, for instance at continental margins or major faults, is included in this effective Q, we believe that the part of Q which is due to absorption of energy is probably higher than that obtained. An estimate of Q from the spectrum of body waves passing virtually vertically through the crust and thence through the mantle has given a value of 1000. It would seem a reasonable hypothesis that the "absorption Q" of rocks is of the order of 1000, but varies considerably with depth. For waves sampling the whole crust scattering at local inhomogeneities can reduce the effective Q to a few hundred while for wave propagating essentially in the upper few kilometers the combined effects of more severe scattering and viscosity associated with interstitial water can reduce the effective Q to be of the order of 100. There is also accumulating some evidence of a decrease of Q in the low velocity layer in the upper mantle. Figure 8 taken from Anderson and Archembeau [1] summarises the available Q data and shows our data superimposed.

1. D. L. Anderson and C. B. Archembeau: (15th May 1964) "The Anelasticity of the Earth". J.G.R., 69, 10, 2071 - 2084

5.4 Spectral Analysis

The generalised expression for the amplitude of a given frequency component has been given in equation (14) as

$$W(w) = Y(w) \times S(w) \times J(w) \times \psi(w).$$

We have shown how Y(w) depends upon the yield of the explosion, i.e.,

$$Y(w) \propto T^{-1}(1 + 40M^{2}T^{-2})^{-1}$$
,

how J(w) depends on the propagation path, i.e.,

$$J(w) \propto UT^{-\frac{1}{2}} (dU/dT)^{-\frac{1}{2}},$$

how $\psi(w)$ depends on the absorption, i.e.,

$$\sqrt{(w)} \propto \exp - (\pi E\Delta/QUT),$$

where, from the amplitude-distance data, our best estimate for O is 340, and we have assumed that S(w) is a constant. However, since W. Y, J and ψ can be estimated it is possible to estimate S(w). Several estimates have been made using the above principle. The method is illustrated in Figure 9 and Figure 10 shows some of the results. In general they seem to indicate that S(w) is constant for T > 30 s but increases quite rapidly for $T \leq 30$ s. Some of the finer detail could be significant but in view of the approximations, and doubts about the shapes of the response curves we feel it unwise to pursue the matter. However, it is worthy of note that, in various seismological laboratories, a considerable amount of theoretical work is being carried out to produce the source function for a layered medium, and comparisons between theory and experiment can therefore be expected in the near future. Intuitively one would expect the source factor to decrease with increasing period because μ increases with depth and the longer waves effectively sample to greater depth. We note that theoretical source functions would then enable Q estimates to be made from individual records by plotting log $W(Y \times S, \times J)$ versus T/U.

Another factor can become important at periods below about 20 s if the group velocity curve shows a minimum. In this case the stationary phase approximation fails to hold and one must use the "Airy" phase approximation. Although evidence of the "Airy" phase was found, the fact that we have used dU/dT means that we believe that our results always apply to periods greater than those at which the minimum occurs. Nevertheless, how close one can get to the "Airy" phase without destroying the validity of the stationary phase approximation is not clear. We consider that the effect would be to increase the observed/predicted amplitude ratio at the shorter periods. It would therefore behave in the same way as we believe the source factor to behave, and be indistinguishable from it.

6. DISPERSION CURVES

As indicated in the previous sections, a requirement for analysis is the derivation from the records of group velocity curves. In our work we have simply taken the location and origin times for any event, as given by the US Coast and Geodetic Survey Provisional Epicentre determinations, and used the group velocity curve thus derived for each station as characteristic of the whole series. No attempt was made to correct for instrument phase response, nor for phase effects at the source. Typical results are shown in Figure 11. It should be emphasised that this figure is intended to show the path differences rather than give absolute data for any specific path. Nevertheless, the data from the atmospheric explosions could be reworked to obtain both phase and group velocity by Brune's method [1] now that an analytical representation of the source function is available.

The most noteworthy detail from the curves is the difference between the mainly continental paths, for instance Novaya Zemlya to Uppsala, and the oceanic paths, for instance Novaya Zemlya to Honolulu. It would also appear that the Novaya Zemlya to Wolverton path indicates a thinner crust on average than the Novaya Zemlya to Uppsala path.

7. SUMMARY AND CONCLUSIONS

A simple theory has been advanced to explain the observed amplitude and character of the seismic surface waves generated by atmospheric explosions. Despite the approximations involved, the predictions are in good agreement with the data. The main difficulties are as follows:-

- (a) The non-linearity of the source is quite pronounced. This principally results in there being a height of burst dependence, but we believe that the effect is relatively unimportant except for the largest yields, say greater than 20 megatons.
- (b) The source factor is assumed to be independent of period. Techniques for taking fully into account the layering are becoming available but, although it will be interested to compare theory and experimental data, the actual computations for these specific cases are not considered
- 1. J.N. Brune, J.E. Nafe and J.E. Oliver: (January 1960) "A Simplified Method for the Analysis and Synthesis of Dispersed Wave Trains". J.G.R., <u>65</u>, 1

worthwhile since the propagation paths are far from uniform.

(c) The instrument calibrations are often not of sufficient quality to permit detailed studies.

The principle achievement in the geophysical domain has been the estimate of values of Q which should prove of considerable use to seismologists since they are obtained in a period range where little data exists. Finally it is apparent that this work provides a foundation for a re-examination of the techniques of deriving magnitudes from surface wave measurements. Briefly we would recommend the use of the 30 s waves and an application of the stationary phase approximation.

8. ACKNOWLEDGMENTS

The authors are grateful to the many people, too numerous to mention individually, who provided the basic seismic data, in the form of long period records. Mr. F.A. Key provided his unpublished results of model scale experiments. Dr. H.N.V. Temperley and Dr.H.I.S. Thirlaway reviewed the manuscript and made helpful suggestions.

APPENDIX

MODEL EXPERIMENTS ON THE GENERATION OF SEISMIC SURFACE WAVES BY ATMOSPHERIC SOURCES

The work described is this Appendix was carried out by Mr. F.A. Key at the Foulness Division of AWRE.

The model used to represent the earth was a concrete block cast into the ground. The block had a plane horizontal surface of dimensions 15×15 ft and its thickness was 3 ft. The concrete mixture was carefully controlled during casting, so that as far as possible its properties were a function of depth only. For the last 6 in. the aggregate used passed through a 1/10 in. sieve. The compressional wave velocity in the block was about 15 000 ft/s.

The source was a spark derived from a condenser of $0.25 \,\mu\text{F}$ charged to 10 kV. Our best estimate of the energy release, obtained from photographic studies of the shock velocity, is 0.3 cal giving a $W^{\frac{1}{3}}$ scaling factor for the source relative to 1 megaton of about 10⁵. The detector was a piezo-electric blender multimorph, feeding conventional amplifiers and recording oscilloscopes.

The first experiment consisted of obtaining records of the surface waves at 10 ft from ground zero for various source heights in the range $\frac{1}{4}$ -3 in. Typical results are shown in Figure 12, and show the theoretically predicted independence of amplitude upon source height.

The second experiment was designed to investigate the effective area of generation. A large $\frac{1}{6}$ in. thick rubber sheet was placed over the central area of the block and it was determined that no detectable surface waves were then generated when the spark was fired. Circular holes of various diameter were then cut in the sheet, so that their centres were directly beneath the spark, and surface wave amplitudes measured for various heights of burst. The results, plotted as amplitude, A, against aperture radius, R, divided by source height, H, are shown in Figure 13. Clearly the generation effectively takes place within a radius equal to the height of burst, the maximum effect, as indicated by dA/d(R/H), occurring at $R/H \approx 0.3$.

It was on this evidence that we decided to take the pressure at a radial distance from the source equal to the height of burst as the effective generating pressure.

		Ma	<u>aqnificati</u>	<u>on Details</u>	Assumed		
<u>Station</u> <u>Name</u>	Code	<u>Co-ordinates</u>	Year	20	Periods	40	Comments
Addis Ababa	AAE	9° 02' N 38 [°] 46' E	1961 1962	840	830	310	WWSS station
Alert	ALE	82 ⁰ 29' N 62 ⁰ 24' W	1961 1962	800 800	850 850	900 900	Canadian station
Albuquerque	ALQ	34 ⁰ 56' N 106 ³ 27' W	1961 1962	1480 2950	1 450 2950	1320 2700	WWSS station
Athens University	ATU	37° 58 [°] N 23 [°] 43' E	1961 1962	1480	1450	- 1320	WWSS station
Blacksburg	BLA	37 ⁰ 15' N 80 ⁰ 25' W	1961 1962	5900	- 5900	5800	WWSS station
Berkeley (Strawberry)	BKS	37° 53' N 122° 14' W	1961 1962	2900	2900	2850	WWSS station
Copenhagen	COP	55° 41' N 12° 26' E	1961 1962	1480	- 1450	- 1320	WWSS station
Florissant	FLO	38° 48' N 90° 22' W	1961 1962	1480	- 1450	1320	WWSS station
Halifax	HAL	44 ⁰ 38' N 63 ⁰ 36' W	1961 1962	690 690	690 690	700 700	Canadian station
Honolulu	HON	21 ⁰ 19' N 158 ⁰ C' W	1961	3500	3300	2800	Good calibrations, calibrated before and during the 1962 series
Kiruna	KIR	67° 50' N 20° 25 E	1961 1962	-	-	- _	Information extracted from Bâth's publication

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			TABL	E 1 (CONT.	Ţ		
Statión	Code	<u>Co-ordinates</u>	Year	20	Periods, 30	s 40	Comments
<u>Name</u> Konsberg	KON	59° 39' N 9° 35' E	1961 1962	1480	1450	1320	WWSS station
London	LDN	43° 02' N 81° 11' W	1961 1962	300	290.	- 280	Canadian station
Lwiro	LWI	2 ⁰ 15' S 28 ⁰ 48' E	1961 1962	1100	1000	850 850	
Minneapolis	NNW	44° 55' N 93° 11' W	1961 1962	- 2990	2990	- 2890	WWSS station
Mould Bay	MBC	76° 14' N 119° 20' W	1961 1962	700	750	780	This instrument was calibrated in Ottawa, re-assembled at MBC station, the magnification is assumed to have remained constant
Madison	SQM	43 ⁰ 22° N 89 ⁰ 46° W	1961 1962	1480 1480	1450	1320	WWSS transient response Fourier analysed – good agreement with WWSS published curve
Montreal	MNT	45° 30' N 73 [°] .37' W	1961 1962	660 660	- 069	720 720	Canadian station
Mt. Trububa	UTM	36 ⁰ 13' N 140 ⁰ 61' E	1961 1962	1100 1400	1200	850 1050	Calibration curves received
Mundaring	MUN	31 ⁰ 59' S 116 ⁰ 12.5'E	1961 1962	740	- 740	- 680	WWSS station
Nurmijarvi	NUR	60° 31' N 24° 39' E	1961 1962	3000	3000	- 2900	WWSS station
Palisades	PAL	41 ⁰ 0' N 73 ⁰ 54' W	1961 1962	1960 1960	1850 1850	1800 1800	Gain obtained from Pomeroy's paper Bull.Seis. Soc. Assn., 53. 1.109 - 149

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			INDLE I	10011.7			
<u>Station</u> Name	<u>Code</u>	<u>Cu-ordinates</u>	Year	20	Periods, 30	s40	Comments
Pasadena	PAS	34 ⁰ 09' N 118 ⁰ 10' W	1961 1962	.10000	8000	3600 -	Acceleration impulse response Fourier analysed
Penticton	PNT	49° 19' N 119° 37' W	1961 1962	940 940	890 890	860 860	Canadian station
Rapid City	RCD	44 ⁰ 04' N 103 ⁰ 12' W	1961 1962	740	740	- 680	WWSS station
Resolute	RES	74 ⁰ 41' N 94 ⁰ 54' W	1961 1962	1050 1050	-	580 580	This station re-calibrated in August 1963 with following results: 20: 600 301 510 40:420 Canadian station
Scarborough	SCB	43 [°] 43' N 79 [°] 14' W	1961 1962	910	950	1020	Canadian station
Schefferville	SCH	54 ⁰ 49' N 66 ⁰ 47' W	1961 1962	1100	1120	1160	Canadian station
State College Pensylvania	SCP	40° 49' N 77° 52' W	1961 1962	740	740	690	WWSS station
Uppsala	UPP	59 ⁰ 51' N 17 ⁰ 37' E	1961 1962	100 1500	-	300 900	
Victoria	VIC	48 ⁰ 31' N 124 ⁰ 25' W	1961 1962	660 660	680 680	700 700	Canadian station
Weston	WES	42° 23' N 71° 19' W	1961 1 96 2	- 1480	1450	1320	WWSS station

TABLE 1 (CONT.)

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			TAB	LE 1 (CONT	<u>.)</u>		
<u>Station</u> Name	Code	<u>Co-ordinates</u>	Year	20	Periods, s 30	40	Comments
Wolverton	WOL	51° 19' N 1° 3' W	1961 1962				Calibrated daily
LRSM STATIONS							
Aurora Wisconsin	ARWS	45 [°] 43' N 88 [°] 9' W]				
Delhi New York	DHNY	42 [°] 14' N 74 [°] 53' W					
Campo California	CPCL	32° 44' N 116° 22' W					
Fillmore Utah	FMUT	39° 13' N ' 112° 12' W					All data from LRSM stations
Las Cruces N. Mexico	LCNM	32 ⁰ 24' N 106 ⁰ 36' W					was extracted from the Geo- technical Corporation Long Bange Seismological Measurements
Mina Nevada	MNNV	38 ⁰ 26' N 118 ⁰ 9' W	(Program. Seismological Bulletin:
Marysville California	MVCL	39 [°] 13' N 121 [°] 18' W					Gains changed daily – stations are calibrated each day
San José Texas	SJTX	27° 36' N98° 19' W					
Taft California	TFCL	35° 10' N 119° 58' W					
Winnemucca Nevada	WINV	41° 21 N 117° 28' W	J				

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TABLE 2

Values of A(I) and B(J) (Sample) for Novaya Zemlya 1961

	20	s Period		40 9	s Period	
Date	B(J)	σ	N	B(J)	٩	N
4th October 6th October 23rd October 30th October Station	1.690 1.781 2.215 2.581	0.041 0.045 0.028 0.045	5 6 7 N	1.393 1.542 2.130 2.530	0.049 0.061 0.026 0.020	4 5 9 6 N
ALE HAL KIR MNT PAL PAS RES UPP VIC WOL	0.000 -0.558 -0.133 -0.403 -0.75 -0.476 0.168 -0.645 -0.430	0.039 0.050 0.002 0.058 0.03 0.058 0.036 0.064 0.039	5 6 2 10 10 11 7 7	0.000 -0.423 -0.633 -0.689 -1.110 -0.230 -0.562 -0.378	0.008 0.058 0.045 0.010 0.010 0.040 0.052 0.074	3 2 3 2 2 10 7 6

Note: For maximum consistency with 1962 add to 20 s B(J) and subtract from A(I) 0.008 add to 40 s B(J) and subtract from A(I) 0.029

-	20 s	Period		40 s Period			
Date	B(J)	σ	N	B(J)	σ	N	
16th September 25th September 27th September 24th December	1.880 2.264 2.268 2.309	0.043 0.041 0.059 0.078	8 14 16 9	1.586 2.218 2.190 2.229	0.055 0.057 0.058 0.085	9 16 23 13	
Station	<u>A(1)</u>	σ	N	A(^1)	σ	<u>N</u>	
ALE ALQ ATU BLA FMUT HAL HON LCNM LDN LWI MBC MNNV MNT PNT RCD RES SCA	0.000 -0.576 -0.324 -0.529 -0.697 -0.384 -1.015 -0.541 -0.383 -0.896 -0.173 -0.530 -0.308	0.038 0.064 0.032 0.073 0.048 0.083 0.018 0.075 0.086 0.089 0.042 0.036 0.045	16 12 9 12 4 11 2 10 13 13 3 16 4	0.000 -0.559 -0.535 -0.523 -0.641 -0.519 -0.434 -0.700 -0.705 -1.096 -0.215 -0.747 -0.685 -0.578 -0.578 -0.431 -0.317 -0:591	0.084 0.019 0.065 0.049 0.186 0.090 0.080 0.048 0.048 0.058 0.058 0.056 0.114 0.056 0.114 0.060 0.027 0.048 0.080	15 32 9 52 12 51 20 53 6 31 4 4	
SCH TSU UPP VIC WINV WOL	-0.169 +0.016 -0.937 -0.452	0.092 0.040 0.089 0.052	6 8 8 13	-0.575 -0.604 -0.258 -0.691 -0.659 -0.357	0.280 0.054 0.065 0.083 0.029 0.064	6 3 10 8 6 14	

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TABLE 3

Values of A(I) and B(J) (Sample) for Novaya Zemlya 1962

TABLE 4

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Summary of Station Data

1962 Series at Novaya Zemlya

eriod	$\frac{U}{T} \frac{3}{2} \left[\frac{dU}{dT} \right]^{-\frac{1}{2}},$ $\frac{U}{cm^{\frac{1}{2}}} s^{-\frac{3}{2}}.$	4882333 330.00 330.00 330.00 330.00 330.00 33333333
40 s P	U, km∕s	
	Station Factor (Rela- tive)	2.00 4.44 4.44 4.44 4.44 4.45 4.55
riod	U3k [dU] ⁻⁵ T3k [dI] ⁵ cm ⁵ s ⁻³ k	33324555 455458 555458 555458 555458 5555
30 s Pe	U, km/s	
	Station Factor (Rela- tive)	7.22 7.52
riod	$\frac{U}{T}^{3} \neq \begin{bmatrix} dU \\ dT \end{bmatrix}^{-1}, \\ cm^{2} s^{-3}/2$	865555650 87.000 87.000 87.000 87.0000 87.0000 87.00000 87.00000000000000000000
20 s Pe	U, km/s	9.22 9.22
	Station Factor (Rela- tive)	2.00 2.00 1.49 1.49 1.49 1.49 1.40 1.40 1.40 1.40 1.40 1.40 1.40 1.40
p	Ed ⁵ sin ⁵ d in 10 ³ km	
	ΔE in 10 ³ km	284.97.96.96.7.9.9.96.9.96.87.9.96.9.7.9.82. 27.2.9.84.4.9.9.9.9.9.9.9.9.9.9.9.9.9.9.9.9.
	Station	ALC ALC ALC ALC ALC ALC ALC ALC ALC ALC



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FIGURE 2. Pr 2 AGAINST DISTANCE



FIGURE 3. SHOWING PREDICTED AMPLITUDE V. YIELD RELATIONSHIP



FIGURE 4. TRANSIENT RESPONSE OF FOUR WWSS OBSERVATORIES



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FIGURE 5. NORMALISED AMPLITUDE V. DISTANCE TO DETERMINE Q

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FIGURE 6. NORMALISED AMPLITUDE V. DISTANCE TO DETERMINE Q

FIGURE 7. NORMALISED AMPLITUDE V. DISTANCE TO DETERMINE Q

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LOGARITHMIC SCALE

AMPLITUDES 0 N

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FIGURE 11. TYPICAL DISPERSION CURVES

HEIGHT OF SOURCE 1 8 in. <u>|</u> 4 in. <u>|</u> 2 in. 0 · 3 m s l in. 2 in. 1.27 m s 4 in. FIGUREI2. HORIZONTAL RAYLEIGH WAVE AMPLITUDE

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AT 10 ft FROM SPARK SOURCE

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FIGURE 13. EFFECTIVE SOURCE SIZE FOR SURFACE WAVES (VERTICAL)